HARP Collaboration

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On cross-section normalization

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Abstract

This note summarizes the steps to take from the number of events in the bin $\Delta p_{\rm T} \Delta \cos\theta$ to the double-differential inclusive cross-section in the bin $\Delta p \Delta \Omega$.

1 What is written on tape?

Be τ the deadtime fraction caused primarily by the data readout cycle (τ = deadtime/(livetime + deadtime); $\tau \approx 0.5$–0.9; readout time per event $\mathcal{O}(1)$ ms).

Be η the probability that a beam particle is accepted as a 'good beam particle'.

Be $f_{\rm sc}$ the scale factor that is applied to 'beam triggers'. Typically, $f_{\rm sc} = 64$, i.e., each 64th beam trigger initiates the readout cycle irrespective of whether the beam trigger led to an 'interaction trigger' or not. Such an event is labelled by a 'downscale bit'.

Be $p_{\rm int}$ the probability that a beam trigger leads to an interaction trigger.

The number of events on tape with a good beam particle and a 'downscale bit' is

$$\sum_i N^i_{\rm beam} \eta (1-\tau)/f_{\rm sc} \; , \tag{1}$$

where the index i runs over all incoming beam particles.

The number of events on tape with a good beam particle and with an 'interaction bit' is

$$\sum_i N^i_{\rm beam} \eta (1-\tau) p_{\rm int} \; . \tag{2}$$

These two sets of events have a common subset: the events which have at the same time a downscale bit and an interaction bit:

$$\sum_i N^i_{\rm beam} \eta (1-\tau) p_{\rm int}/f_{\rm sc} \; . \tag{3}$$

The total number of events on tape with a good beam particle is

$$\sum_i N^i_{\rm beam} \eta (1-\tau) [p_{\rm int} + (1-p_{\rm int})/f_{\rm sc}) \tag{4}$$

The comparison of the number of events on tape with both the downscale bit and the interaction bit (Eq. 3) with the number of events with the downscale bit (Eq. 1) permits the measurement of the scaling factor $f_{\rm sc}$. Analogously, the comparison with the number of events with the interaction bit (Eq. 2) permits the measurement of the probability $p_{\rm int}$ of the interaction trigger.

For the cross-section normalization, the observed number of interaction events (after due corrections for inefficiencies) is related with the number of good beam particles impinging on the target.

The definition of a good beam particle must be exactly the same for the determination of the number of interaction events and of the number of beam particles. Only in this case the probability η will cancel and thus be of no importance.

In this context, it is important to note that there are two types of downscale beam triggers written to tape: 'DOWNSCALE STROBE' and 'DOWNSCALE BEAM'. Since

‘BEAM’ = ‘STROBE .AND. TDS .AND. HALO_VETO’ and
‘PHYSICS’ = ‘BEAM .AND. (ITC .OR. FTP)’,

it follows that only events with the ‘DOWNSCALE BEAM’ bit are to be used for normalization, otherwise the definition of a ‘good beam particle’ would be different when counting interaction events and beam particles.

We note further that the deadtime fraction τ also cancels and thus is of no importance.

2 Cross-section normalization

We consider the specific example of cross-section normalization for a thin beryllium target.

What we want is the double-differential inclusive cross-section of particle production per target nucleus, per GeV/c of momentum, and per steradian, defined through

$$\Delta N_{\rm trk} = N_{\rm beamp} \frac{N_{\rm A}}{A} \rho d \frac{{\rm d}^2\sigma}{{\rm d}p{\rm d}\Omega} \Delta p \, \Delta\Omega \; , \tag{5}$$

where

- $\Delta N_{\rm trk}$ is the number of tracks counted in the respective bin of momentum and solid angle;
- $N_{\rm beamp}$ is the effective number of beam particles impinging on the target (‘effective’ because of the attenuation correction, see below);
- $N_{\rm A} = 6.022 \times 10^{23}$ mol^{-1} is Avogadro’s constant;
- A is the atomic number ($A = 9.012$ for beryllium);
- ρ is the density in g/cm^3 ($\rho = 1.848$ g/cm^3 for beryllium);
- d is the target thickness in cm ($d = 2.03$ cm for the thin beryllium target);
- ${\rm d}^2\sigma/{\rm d}p{\rm d}\Omega$ is the double-differential inclusive cross-section in mb/(GeV/c sr);
- Δp is the momentum bin in which the tracks are counted;
- $\Delta\Omega$ is the bin of solid angle in which the tracks are counted;
- $\Delta N_{\rm trk}$ is determined from all events with an interaction bit, irrespective of whether they have a downscale bit or not; and
- $N_{\rm beamp} = \sum_i N^i_{\rm beam} \eta (1 - \tau)$ is determined according to Eq. (1) by the number of events with a downscale bit, irrespective of whether they have an interaction bit, multiplied with the scale factor $f_{\rm sc}$.

The incoming particle beam is exponentially attenuated inside the target according to

$$\exp(-\frac{\sigma N_{\rm A}}{A}\rho d) \ , \tag{6}$$

where d is the thickness of the target in g/cm^2. This attenuation leads to a smaller 'effective' number of beam particles which is obtained by multiplying the number of incoming beam particles with the correction factor

$$f_{\rm att} = \frac{1}{\frac{\sigma N_{\rm A}}{A}\rho d} \cdot \left(1 - \exp(-\frac{\sigma N_{\rm A}}{A}\rho d)\right) \ . \tag{7}$$

Numerically, $f_{\rm att} = 0.9754$ for a 5% $\lambda_{\rm abs}$ target.

We have a selection of 'good' TPC sectors: we discard tracks from the 'horizontal' sectors 2 and 5 because:

1. they are badly affected by missing pads which impairs the quality of track reconstruction and also, for the loss of 'useful' RPC overlaps, the reliability of dynamic track distortion corrections;
2. they are badly affected by crosstalk; and
3. no viable cross-checks from cosmic-muon tracks are available (the statistics of horizontal cosmic muons are too low).

A further cut in the azimuthal angle ϕ is applied to avoid the dead regions of the six 'spokes' that subdivide the TPC pad plane into six sectors: 10° on one side and 2° on the other side of each spoke for tracks of one charge, and *vice versa* for the other charge.

Altogether, this reduces the ϕ acceptance by a factor of $\frac{4}{6}\frac{48}{60}$.

With the exception of cuts in the azimuthal and polar angles, event losses from selection cuts, and migration effects from finite resolutions of measured quantities, are assumed to have been corrected by appropriate factors before calculating the double-differential cross-section.

Finally, we must take into account that our events are not collected in bins of momentum p and solid angle Ω, but in bins of transverse momentum $p_{\rm T}$ and polar angle θ (for the binning, see Section 3).

All this leads to the following formula for the calculation of the double-differential inclusive cross-section $\mathrm{d}^2\sigma/\mathrm{d}p_{\rm T}\mathrm{d}\cos\theta$ for particle production from a 5% $\lambda_{\rm abs}$ beryllium target:

$$\begin{aligned}
\frac{\mathrm{d}^2\sigma}{\mathrm{d}p_{\rm T}\mathrm{d}\cos\theta} &= 2\pi \, \frac{A}{N_{\rm A}}\frac{1}{\rho d}\frac{\Delta N_{\rm trk}}{\Delta p_{\rm T}\Delta\Omega}\frac{1}{f_{\rm att}N_{\rm beamp}} \\
&= 1\times 10^{27}\frac{9.012}{6.022\times 10^{23}}\frac{1}{1.848\cdot 2.03}\times \\
&\quad \frac{\Delta N_{\rm trk}}{(p_{\rm T\,max}-p_{\rm T\,min})\,(\frac{4}{6}\frac{48}{60})\,(\cos\theta_{\rm min}-\cos\theta_{\rm max})}\cdot\frac{1}{0.9754\,N_{\rm beamp}} \\
&= 7.668\times 10^{3}\cdot\frac{\Delta N_{\rm trk}}{(p_{\rm T\,max}-p_{\rm T\,min})\,(\cos\theta_{\rm min}-\cos\theta_{\rm max})}\cdot\frac{1}{N_{\rm beamp}} \ ,
\end{aligned}$$

where $p_{\rm T}$ is expressed in GeV/c. The relation with the wanted cross-section $\mathrm{d}^2\sigma/\mathrm{d}p\mathrm{d}\Omega$ is:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}p\mathrm{d}\Omega} = \frac{\langle \sin\theta \rangle}{2\pi} \cdot \frac{\mathrm{d}^2\sigma}{\mathrm{d}p_{\rm T}\mathrm{d}\cos\theta} , \qquad (8)$$

where $\langle \sin\theta \rangle$ is averaged over the respective polar-angle bin, and $\mathrm{d}^2\sigma/\mathrm{d}p\mathrm{d}\Omega$ is expressed in mb/(GeV/c sr).

3 Binning in $p_{\rm T}$ and θ

In Tables 1 and 2 we give the binning in the transverse momentum $p_{\rm T}$ and in the polar angle θ, respectively, as used in our analysis:

Table 1: Binning binning in the transverse momentum $p_{\rm T}$.

Bin number	$p_{\rm T}$ range [GeV/c]
1	0.10 – 0.13
2	0.13 – 0.16
3	0.16 – 0.20
4	0.20 – 0.24
5	0.24 – 0.30
6	0.30 – 0.36
7	0.36 – 0.42
8	0.42 – 0.50
9	0.50 – 0.60
10	0.60 – 0.72
11	0.72 – 0.90
12	0.90 – 1.25

Table 2: Binning binning in the polar angle θ.

Bin number	θ range [deg]
1	20 – 30
2	30 – 40
3	40 – 50
4	50 – 60
5	60 – 75
6	75 – 90
7	90 – 105
8	105 – 125